Dynamic Inverse Scattering

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WIAS

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Dynamical Inverse Scattering, Survey

1. Static scatterer and wave, i.e. one frequency time-harmonic wave
2. Multi-Frequency scattering, static scatterer
3. Dynamical wave field, i.e. time-dependent pulse
4. Moving Scatterer, i.e. constant speed, accelerating, rotating
5. Scatterer is evolving, i.e. changing its location or shape, we get repeated measurements for various time-slices
6. Inverse Scattering Problem as part of a larger dynamic inverse problem.

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DWD

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Outline

Orthogonality Sampling Method

ALGORITHM (ONE-WAVE OS, MULTI-WAVE OS)

For fixed wave number κ one-wave orthogonality sampling calculates

$$\mu(y,\kappa) = \left| \int_{\mathbb{S}} e^{i\kappa\hat{\varphi}\cdot y} u^{\infty}(\hat{\varphi})\, ds(\hat{\varphi}) \right| \quad (1)$$

on a grid $\mathcal{G}$ of points $\tilde{y} \in \mathbb{R}^m$ from the knowledge of the far field pattern u^{∞} on the unit sphere $\mathbb{S}$.

For fixed wave number κ multi-direction orthogonality sampling calculates

$$\mu(y,\kappa) = \int_{\mathbb{S}} \left| \int_{\mathbb{S}} e^{i\kappa\hat{\varphi}\cdot y} u^{\infty}(\hat{\varphi},\theta)\, ds(\hat{\varphi}) \right| ds(\theta) \quad (2)$$

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Multi-frequency Orthogonality Sampling

ALGORITHM (MULTI-FREQUENCY)

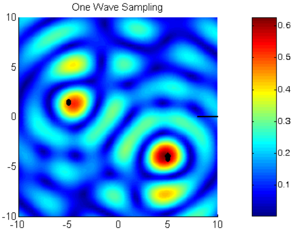
The multi-frequency orthogonality sampling calculates

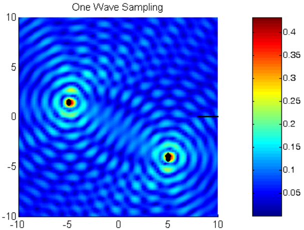
$$\mu(y,\theta) = \int_{\kappa_0}^{\kappa_1} \left| \int_{\mathbb{S}} e^{i\kappa\hat{\varphi}\cdot y} u^{\infty}(\hat{\varphi},\theta)\, ds(\hat{\varphi}) \right| d\kappa \qquad (3)$$

on a grid $\mathcal{G}$ of points $\tilde{y} \in \mathbb{R}^m$ from the knowledge of the far field pattern $u_{\kappa}^{\infty}(\hat{\varphi})$ for $\hat{\varphi} \in \mathbb{S}$ and $\kappa \in [\kappa_0, \kappa_1]$.

Here also multi-direction multi-frequency sampling is possible by adding the indicator functions for several directions of incidence.

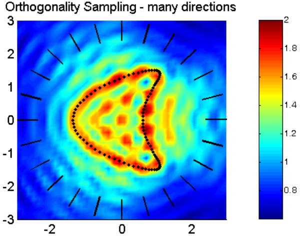
One Wave, one frequency: the simplest setting

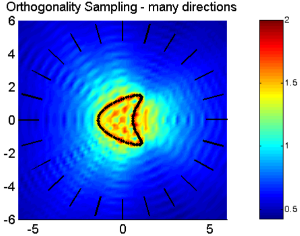




Graphics: Orthogonality sampling with $\kappa = 1$ or $\kappa = 3$ for fixed frequency, one direction of incidence

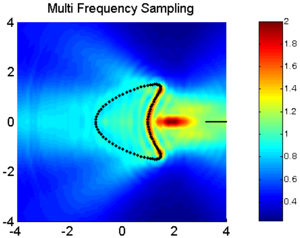
Multi-direction Ortho Sampling

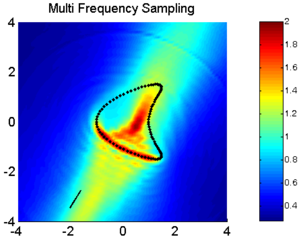




Graphics: Orthogonality sampling, many directions of incidence, fixed frequency

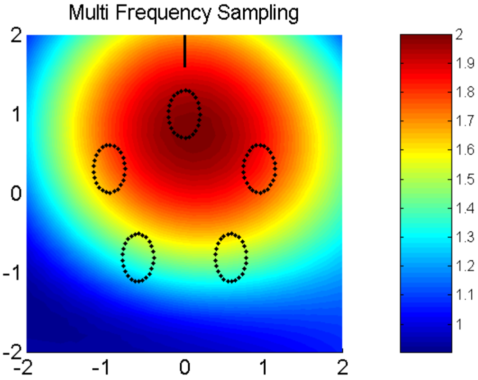
Multi-frequency Ortho Sampling





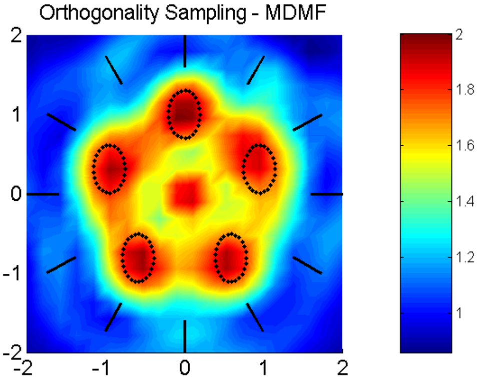
Graphics: Orthogonality sampling, many directions of incidence, fixed frequency

Resolution Study: Large Scale



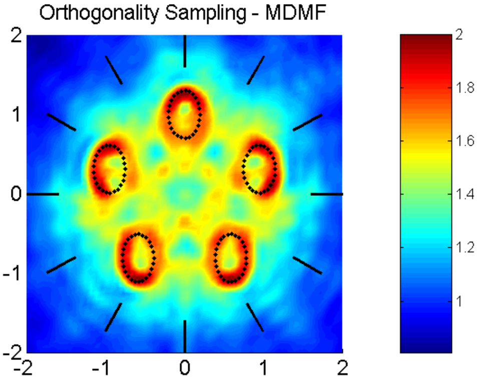
Graphics: Multi-frequency Orthogonality sampling with κ between 0.1 and 1, i.e. with a frequency between $\lambda = 6$ and $\lambda = 60$, one direction of incidence

Resolution Study: Medium Scale



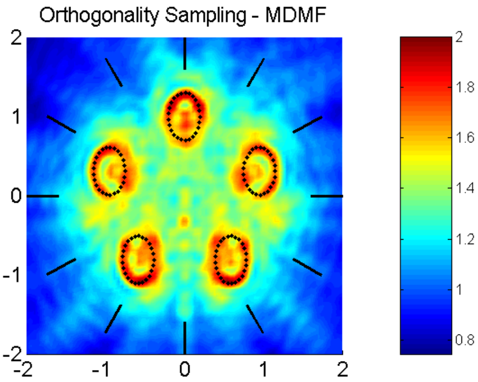
Graphics: MDMF Orthogonality sampling with κ between 3 and 4, i.e. with a frequency between $\lambda = 1.5$ and $\lambda = 2$

Resolution Study: Medium Scale



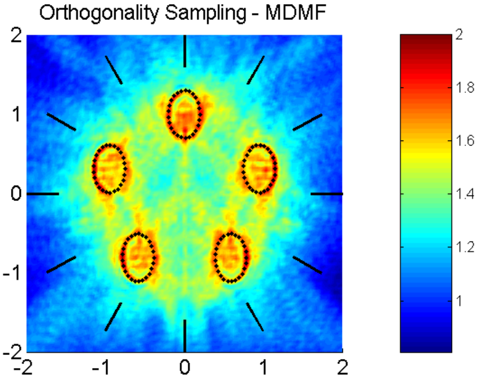
Graphics: MDMF Orthogonality sampling with κ between 6 and 15, i.e. with a frequency between $\lambda = 0.4$ and $\lambda = 1$

Resolution Study: Fine Scale



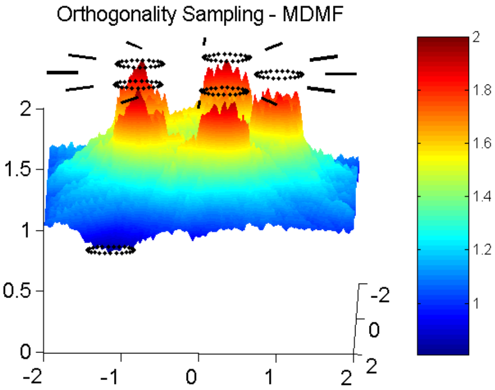
Graphics: MDMF Orthogonality sampling with κ between 10 and 20, i.e. with a frequency between $\lambda = 0.3$ and $\lambda = 0.6$

Resolution Study: Very Fine Scale



Graphics: MDMF Orthogonality sampling with κ between 20 and 40, i.e. with a frequency between $\lambda = 0.15$ and $\lambda = 0.3$

Resolution Study: Very Fine Scale



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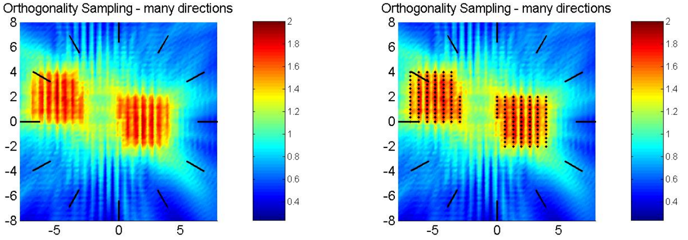
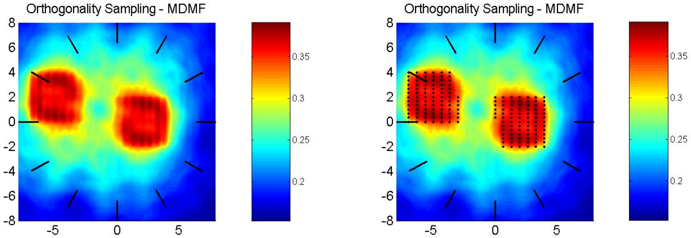
Medium Reconstructions I



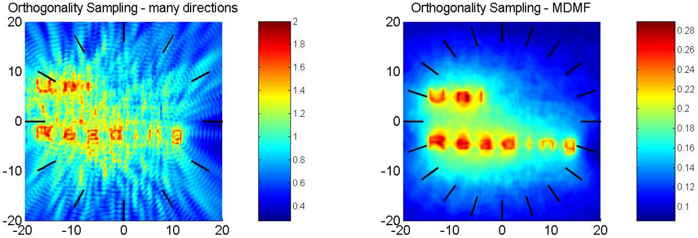
Graphics: Orthogonality sampling for medium reconstruction, MD, fixed frequency $\kappa = 9$.

Medium Reconstructions II



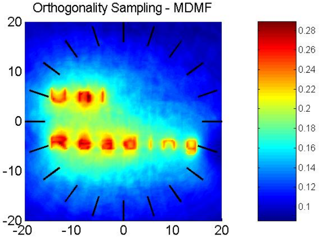
Graphics: Orthogonality sampling for medium reconstruction, MDMF.

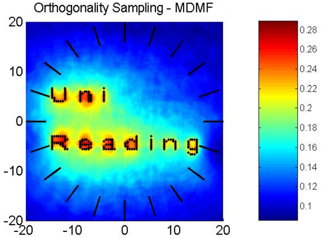
Medium Reconstructions III



Graphics: Orthogonality sampling for medium reconstruction, MDMF.

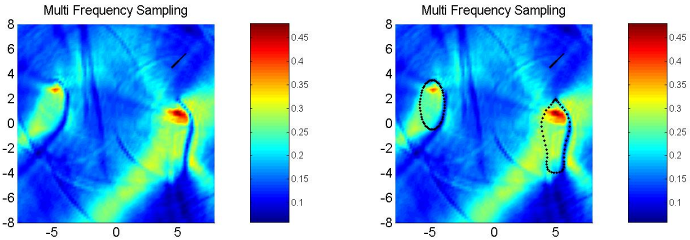
Medium Reconstructions IV





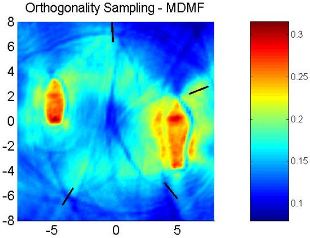
Graphics: Orthogonality sampling for medium reconstruction, MDMF.

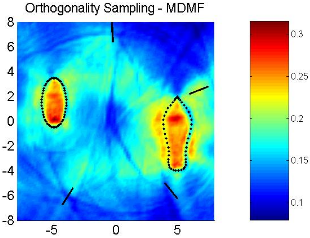
Neumann BC I



Graphics: Orthogonality sampling for the Neumann BC, MF.

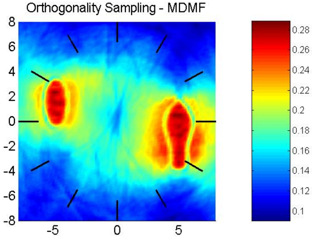
Neumann BC II

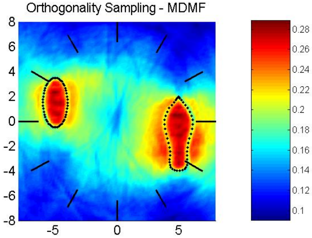




Graphics: Orthogonality sampling for the Neumann BC, MDMF.

Neumann BC II





Graphics: Orthogonality sampling for the Neumann BC, MDMF.

Orthogonality Sampling Convergence Dirichlet Case

Theorem (Convergence or Ortho-Sampling, P 2007/08)

The orthogonality sampling algorithm with the Dirichlet boundary condition for one-wave fixed frequency reconstructs the reduced scattered field, i.e.

$$u^s_{red}(x) = \int_{\partial D} j_0(\kappa|x-y|)\frac{\partial u(y)}{\partial \nu(y)}\, ds(y), \quad x \in \mathbb{R}^m. \tag{4}$$

Convergence analysis of the method can be based on the Funk-Hecke formula.

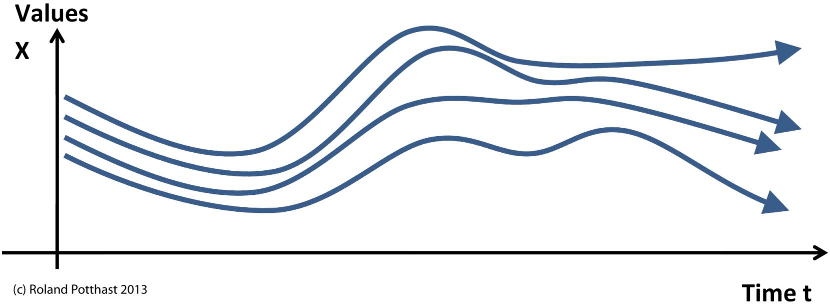
Literature

- Potthast, R.: Acoustic Tomography by Orthogonality Sampling, Institute of Acoustics Spring Conference, Reading, UK 2008.
- Potthast, R: Orthogonality Sampling for Object Visualization, Inverse Problems 2010.
- Griesmaier, R: Multi-frequency orthogonality sampling for inverse obstacle scattering problems, Inverse Problems (2011)

Outline

Outline

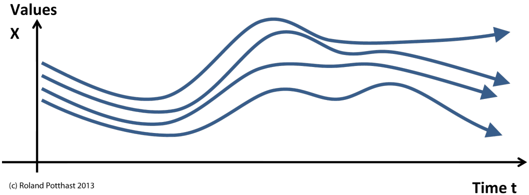
A dynamical system



- We have some **state space** X with states φ.
- We have some **dynamics** M mapping $\varphi(s)$ into $\varphi(t)$ for $t \geq s \in \mathbb{R}$:

$$\varphi(t) = M(s, t, \varphi(t)), \quad t \geq s \in \mathbb{R}. \tag{5}$$

A dynamical system



M can be given by some **differential equation** or system of ODE:

$$\dot{\varphi}(t) = F(t, \varphi(t)), \quad t \geq 0 \tag{6}$$

with **initial condition**

$$\varphi(0) = \varphi_0. \tag{7}$$

We can solve these systems by standard tools as described in lectures about ODE, e.g. the Runge-Kutta Method.

A dynamical system

Often, X is a **normed space** or **Hilbert space**, each state $\varphi(t)$ is a function on some domain Ω, i.e.: $\varphi(t) = \{\varphi(x,t) : \ x \in \Omega\}$ for $t \geq 0$.

Dynamical PDE System

The dynamical system of nonlinear partial differential equations has the form

$$\dot{\varphi}(x,t) = F(t, x, \varphi(x,t)), \quad x \in \Omega, t \geq 0 \tag{8}$$

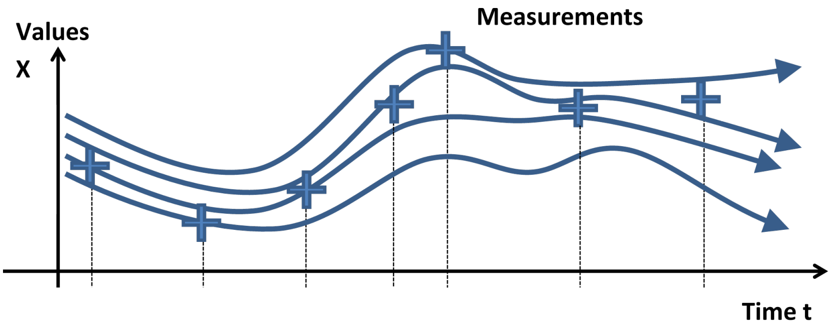
with **initial conditions** (IC)

$$\varphi(x,t) = \varphi_0(x), \quad x \in \Omega \tag{9}$$

and **boundary conditions** (BC)

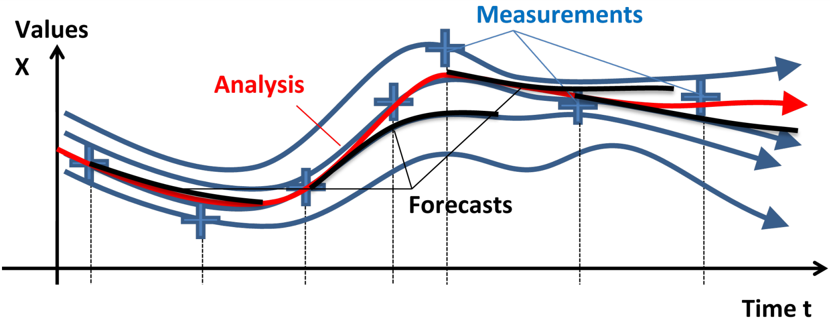
$$\varphi(x,t) = \psi(x,t), \quad x \in \partial\Omega, \quad t \geq 0. \tag{10}$$

Main Task of Data Assimilation I



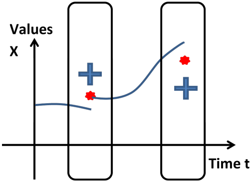
- We measure data $f_k \in Y$ at time $t_k \geq 0$ in an **observation space** Y.
- The task of **data assimilation** is to employ measured data f_k at time t_k to control the dynamical system $\varphi(t)$ and provide realistic states $\varphi^{(a)}(t)$, also called *the analysis*.

Main Tasks Data Assimilation II

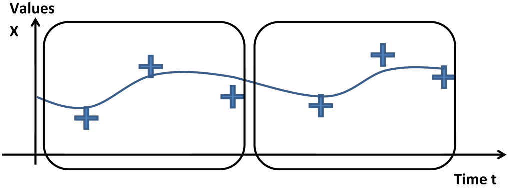


- Provide an **estimate for the whole state** $\varphi \in X$, even if parts of it cannot be measured.
- Calculate initial conditions for **forecasts**.
- Determine a coherent trajectory over time, when data assimilation is recalculated with one coherent DA system, to study the state evolution. This is called **reanalysis**.

Treatment of the Temporal Dimension

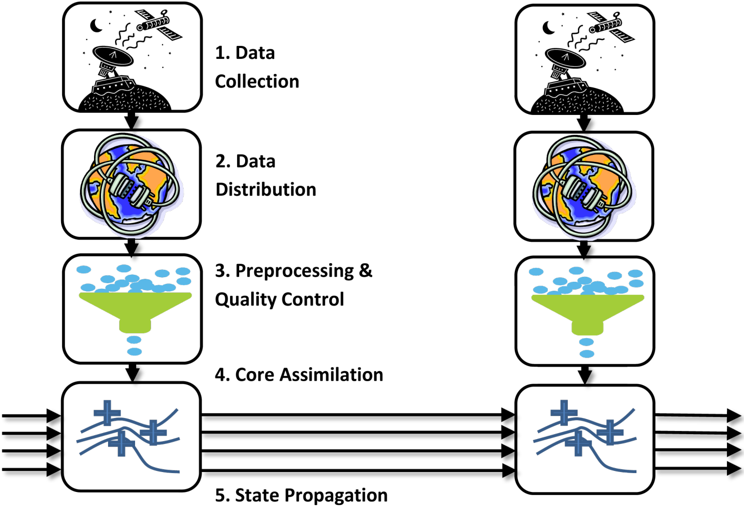


(3dVar)



(4dVar)

The Data Assimilation Process



Motivation I

Let H be the observation operator mapping the state φ onto the measurements f. Then we need to update or find φ using the equation

$$H(\varphi) = f,$$

where H^{-1} is unstable or unbounded. When we have some initial guess $\varphi^{(b)}$, we transform the equation into

$$H(\varphi - \varphi^{(b)}) = f - H\varphi^{(b)},$$

with the incremental form

$$\varphi = \varphi^{(b)} + H^{-1}(f - H\varphi^{(b)}).$$

Least Squares

In order to find out φ we should minimize the functional

$$J(\varphi) := \left\|\varphi - \varphi^{(b)}\right\|^2 + \left\|f - H\varphi^{(b)}\right\|^2.$$

The normal equations are obtained from first order optimality conditions

$$\nabla_{\varphi} J = 0.$$

Usually, the relation between variables at different points is incorporated by using covariances/weighted norms:

$$J(\varphi) := \left\|\varphi - \varphi^{(b)}\right\|^2_{B^{-1}} + \left\|f - H\varphi^{(b)}\right\|^2_{R^{-1}},$$

The variational update formula is now

$$\varphi^{(a)} = \varphi^{(b)} + BH^*\left(R + HBH^*\right)^{-1}\left(f - H\varphi^{(b)}\right)$$

Kalman Filter

In the Kalman filter method we calculate an analysis update by

$$\varphi_k^{(a)} = \varphi_k^{(b)} + B_k^{(b)} H^* \big(R + HB^{(b)} H^*\big)^{-1} \big(f_k - H\varphi_k^{(b)}\big) \tag{11}$$

and an covariance update by

$$B_k^{(a)} = (I - KH) B_k^{(b)}, \quad k = 1, 2, 3, \ldots \tag{12}$$

with the *Kalman Gain Matrix*

$$K_k = B_k^{(b)} H^* \big(R + HB_k^{(b)} H^*\big)^{-1}$$

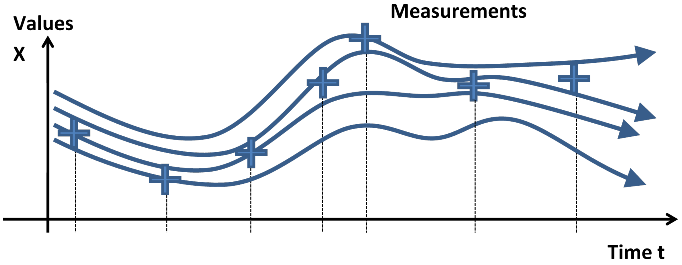
and the weight or covariance matrix B evolves with the model dynamics M,

$$B_{k+1}^{(b)} = M_k B_k^{(a)} M_k^*, \quad k = 1, 2, 3, \ldots \tag{13}$$

Kalman Filter for Large-Scale Problems?

1. In Numerical Weather Prediction (NWP) the typical problem size is around $n = 10^8$ unknowns and could easily be larger when resolution is increased. The number of measurements which are employed at each time t_k are around $m = 10^7$.
2. In the Kalman Filter, this would lead to matrices B of the size $10^8 \times 10^8$, which has strong impact on calculation times.
3. For short range numerical weather prediction (SRNWP), we have only around 15 minutes on a supercomputer to calculate the analysis, for modern applications with fast update rates we need to go down to 5min.
4. One main problem of modern NWP is to find low-dimensional approximations which can be incorporated into the algorithms!

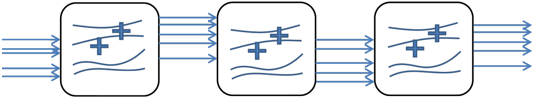
Use Ensembles for Approximation



- Instead of running only one version of our dynamical system, we run L different versions of it, which we call **ensembles** or **particles**.
- This is computationally expensive for the forward problem, but we will save on the minimization needed for calculating the analysis.
- With the ensemble we can **capture the uncertainty** both in the model as well as in the analysis!

Outline

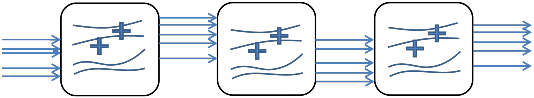
Ensemble Kalman Filter



The main idea of the **Ensemble Kalman Filter** is to approximate the B matrix in all of its steps by an ensemble in the form $B = QQ^*$, when

$$Q := \frac{1}{\sqrt{L-1}}\left(\varphi^{(1)} - \mu, ..., \varphi^{(L)} - \mu\right)$$

with ensemble mean $\mu = \sum_{j=1}^{L} \varphi^{(j)}$. This is the standard **unbiased stochastic estimator** for the covariance matrix.



We need to propagate the ensemble through time. Starting with an ensemble $\left\{\varphi_0^{(l)}, \quad l = 1, ..., L\right\}$, this leads to ensemble members

$$\varphi_{k+1}^{(l)} = M_k \varphi_k^{(l)}, \quad k = 1, 2, 3, ...$$

This means that we solve the equation in a low-dimensional subspace

$$U^{(L)} := \text{span}\{\varphi_k^{(1)} - \mu_k, ..., \varphi_k^{(L)} - \mu_k\}.$$

The **update formula** now is

$$\varphi_k^{(a)} = \varphi_k^{(b)} + Q_k Q_k^* H^* \left(R + H Q_k Q_k^* H^*\right)^{-1} \left(f_k - H\varphi_k^{(b)}\right)$$

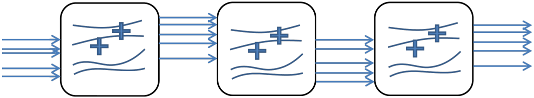
The updates of the EnKF are a linear combination of the columns of Q_k. We can therefore write

$$\varphi_k - \varphi_k^{(b)} = \sum_{l=1}^{L} \gamma_l \frac{1}{\sqrt{L-1}} \left(\varphi_k^{(l)} - \overline{\varphi}_k^{(b)}\right) = Q_k \gamma$$

with coefficient vector $\gamma \in \mathbb{R}^L$. The resulting the expresion to minimize is

$$J(\gamma) := \|Q_k\gamma\|^2_{B_k^{-1}} + \|f_k - H\varphi_k^{(b)} - HQ_k\gamma\|^2_{R^{-1}}.$$

Ensemble Kalman Filter: Summary



In the Ensemble Kalman filter method we calculate an analysis update by

$$\varphi_k^{(a)} = \varphi_k^{(b)} + Q_k^{(b)} Q_k^{(b),*} H^* \left(R + H Q_k^{(b)} Q_k^{(b),*} H^* \right)^{-1} \left(f_k - H \varphi_k^{(b)} \right) \quad (14)$$

and a covariance update by $Q_k^{(a)} = Q_k^{(b)} S$ with $S \in \mathbb{R}^{L \times L}$ given by

$$S = \sqrt{ I - (Q_k^{(b)})^* H_k^* \left(R + H Q_k^{(b)} (Q_k^{(b)})^* H_k^* \right)^{-1} H_k Q_k^{(b)} } \quad (15)$$

and the ensemble $\{\varphi^{(1)}, \ldots, \varphi^{(L)}\}$ evolves with the model dynamics M by,

$$\varphi_{k+1}^{(b,\ell)} = M_k \varphi_k^{(a,\ell)}, \quad \ell = 1, \ldots, L, \; k = 1, 2, 3, \ldots \quad (16)$$

Outline

Least Squares Analysis Model

To *understand the role of localization*, we study a simplified problem which is characteristic for our analysis step in the EnKF.

- One dimensional model without cycling
- **Least square estimation** to obtain the analysis (LSA) and the truth is given by a high-order function.
- The analysis is obtained using both all available observations and only a **local set**.
- Estimation performed with and without **background terms**.
- Observations are generated from the truth with a specified **observation error** σ_{obs}.
- Analysis approximated by straight lines $a + bx$ (an ensemble of linear functions).

Example 1a

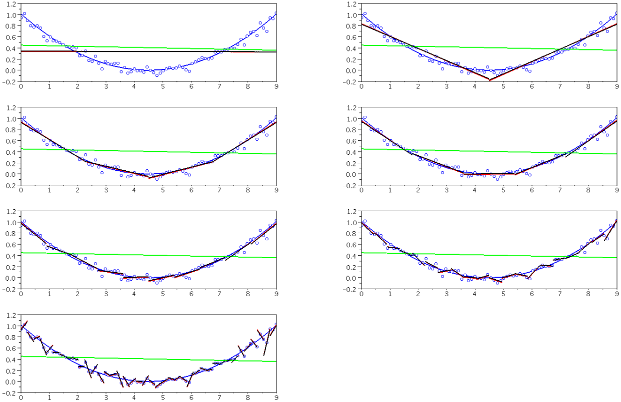


Fig.1: Truth (blue line), observations (blue circles), background (green), no background LSA (red) and background LSA (black) for $\sigma_{obs} = 0.05$ and different localization radii.

Example 1b

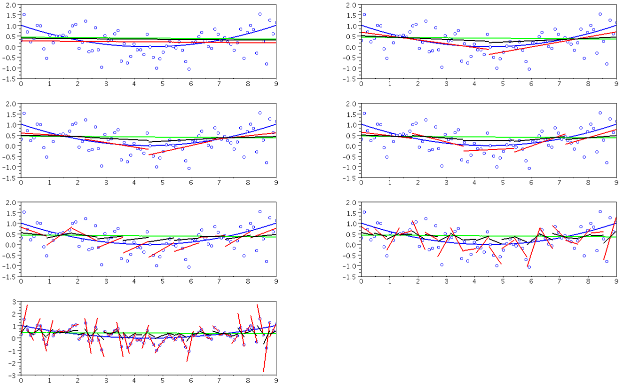


Fig.2: Truth (blue line), observations (blue circles), background (green), free LSA (red) and bg LSA (black) for $\sigma_{obs} = 0.5$ and different localization radii.

Remarks

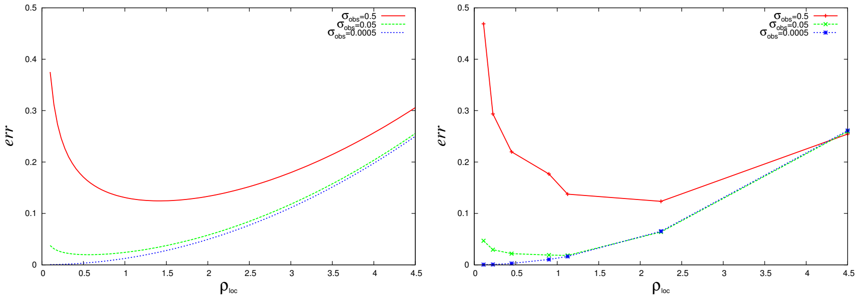
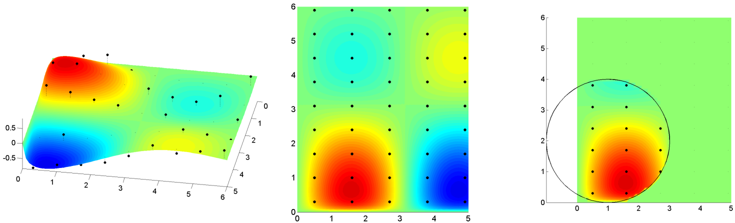


Fig.4: Theoretical and numerical results for error as a function of ρ_{loc}, $\sigma_{obs} = [0.0005\ 0.05\ 0.5]$.

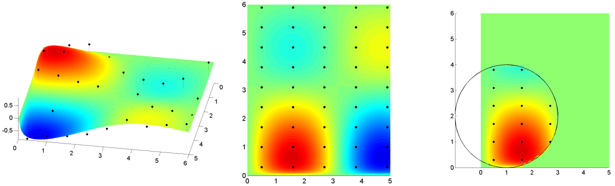
- The optimal value of ρ_{loc} takes smaller values when σ_{obs} decreases.
- For large values of σ_{obs} the analysis without the background correction is clearly worse than analysis considering the background.

Idea of Localization I



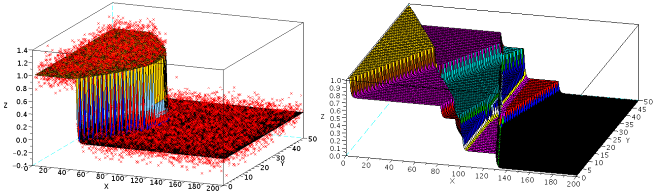
- Carry out the ensemble analysis in subsets of the full spatial domain!
- Given a **localization radius** $\rho > 0$ the analysis at a point x this is effectively using only observations at one point y with $\|x - y\| \le \rho$.

Different Forms of Localization

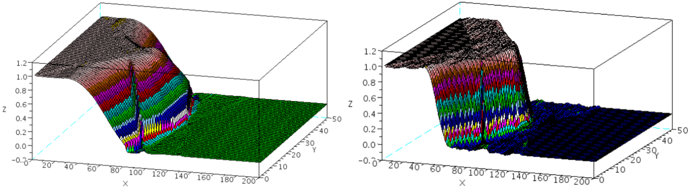


- We can localize the B matrix by multiplying it element-wise with some **localization matrix** C, i.e. taking the **Schur product** $B \bullet C$.
- In this case, if the localized B-matrix does not have any particular **block structure**, localization still involves all variables!
- We can **localize the observations**, by carrying out an analysis with a limited set of observations located in some domain D. But then **we also need the localization of the background term**, since otherwise remote features of the background might dominate the local analysis at the observation point.

Example 2a: What Localization Achieves



Truth (left) and solution by EnKF with straight front ensemble without localization.



Solution by EnKF with straight front ensemble with medium (left) and strong (right) localization.

Outline

Outline

Recall our Setup

We start with the update formula

$$\varphi^{(a)} = \varphi^{(b)} + BH'\big(R + HBH'\big)^{-1}\big(f - H\varphi^{(b)}\big).$$

In the EnKF methods the background convariance matrix is represented by $B_k^{(ens)} := Q_k Q_k^*$, where the ensemble matrix Q_k is defined as

$$Q_k := \frac{1}{\sqrt{L-1}} \left(\varphi_k^{(1)} - \overline{\varphi}_k^{(b)}, ..., \varphi_k^{(L)} - \overline{\varphi}_k^{(b)} \right),$$

where $\overline{\varphi}^{(b)}$ denotes the mean $\frac{1}{L}\sum_{l=1}^{L} \varphi^{(l)}$.

Thus, we solve the update in a low-dimensional subspace

$$U^{(L)} := \text{span}\{\varphi_k^{(1)} - \overline{\varphi}_k^{(b)}, ..., \varphi_k^{(L)} - \overline{\varphi}_k^{(b)}\}.$$

EnKF, Coefficients and Norms

The EnKF update formula now is

$$\varphi_k^{(a)} = \varphi_k^{(b)} + Q_k Q_k^T H^* \big(R + H Q_k Q_k^T H^*\big)^{-1} \big(f_k - H\varphi_k^{(b)}\big)$$

The updates of the EnKF are a linear combination of the columns of Q_k. We can therefore write

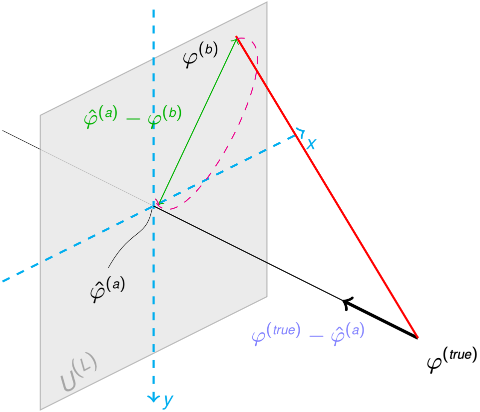
$$\varphi_k^{(a)} - \varphi_k^{(b)} = \sum_{l=1}^{L} \gamma_l \left(\varphi_k^{(l)} - \overline{\varphi}_k^{(b)} \right) = Q_k \gamma$$

in the subspace $U^{(L)}$. We study the **analysis error** in the norm

$$E_k := \big\| \varphi_k^{(a)} - \varphi_k^{(true)} \big\|_{H^* R^{-1} H}, \tag{17}$$

where for simplicity we will assume that H is injective throughout here.

Geometric View



Complete Local EnKF Error Analysis

Theorem (Local EnKF Error Analysis)

The analysis error for the localized Ensemble Kalman Filter is estimated by

$$\|\varphi^{(a)} - \varphi^{(true)}\|_{H^* R^{-1} H} \leq \|R_\alpha\|\delta + E^{(b)}\sqrt{q_k^2 + (1 - q_k^2)c\rho^2} \quad (18)$$

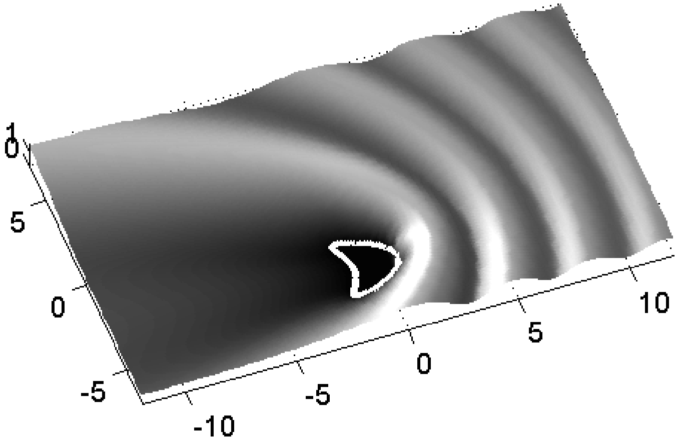
with some constant $q_k < 1$ *and* $E^{(b)} = \|\varphi^{(b)} - \varphi^{(true)}\|$.

$$\begin{aligned} \|\varphi^{(a)} - \varphi^{(true)}\| &\leq \|\varphi^{(a)} - \tilde{\varphi}^{(a)}\| + \|\tilde{\varphi}^{(a)} - \varphi^{(true)}\| \\ &\leq \|R_\alpha\|\delta + E_k. \end{aligned} \quad (19)$$

Details can be found in Perianez, P. and Reich: *Error Analysis and Adaptive Localization for Ensemble Methods in Data Assimilation*, Preprint.

Outline

Scattering



Dynamic Inverse Problem: Moving Scatterer

- Moving Scatterer
- Wave scattering at times t_k, $k = 1, 2, 3, \ldots$, temporal scales separated!
- Measurements of the far field patterns u_k^∞ at time t_k.
- Task: Track Location of the Scatterer
- Systems M: dynamics is movement to the right with unknown v_2-component of the speed v, only known approximately!
- For numerical example: form of scatterer known, local inversion using the point source method (P. 1996) or Kirsch-Kress method (1986)

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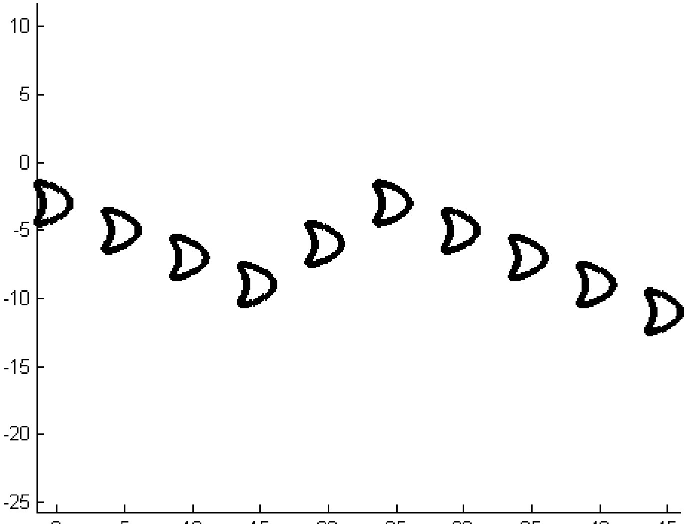
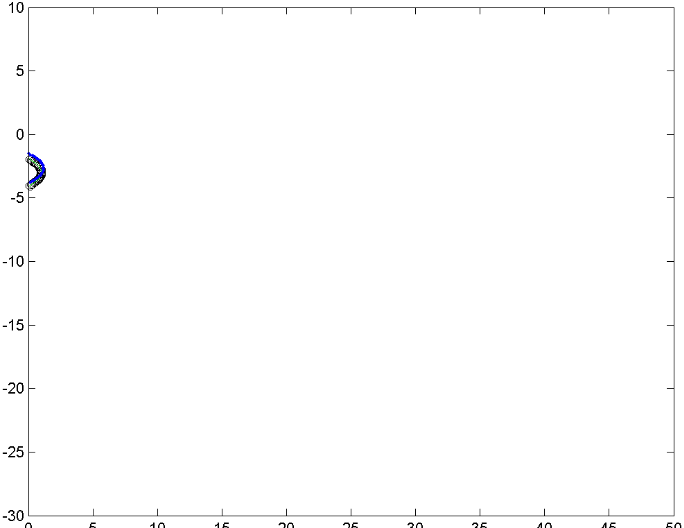
Original Movement



First Guess and Reconstruction



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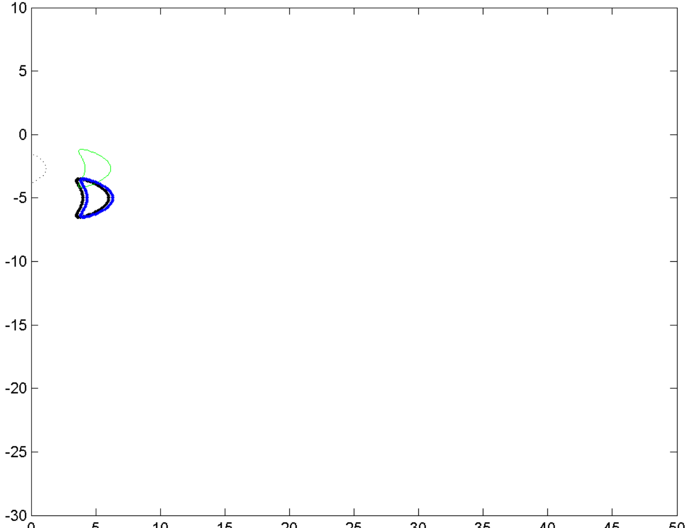

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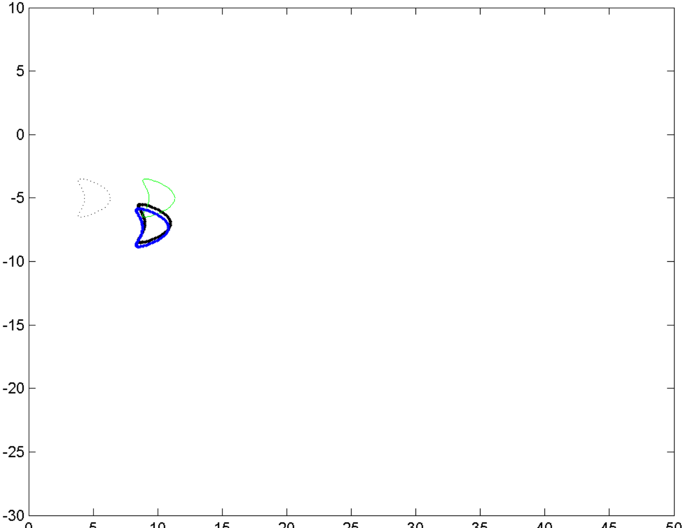

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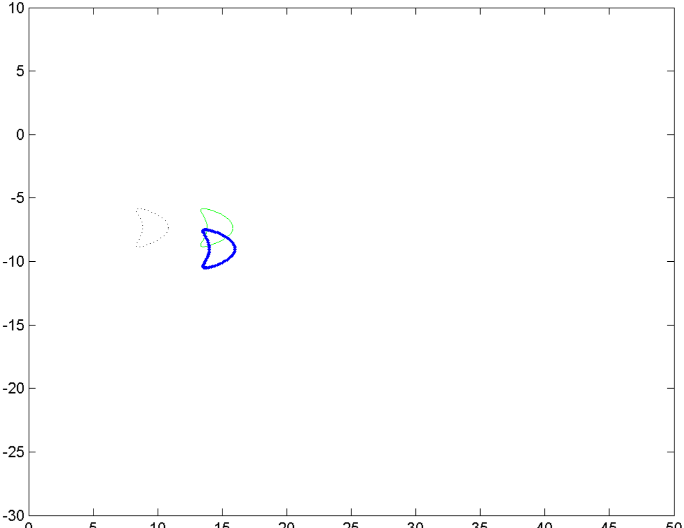

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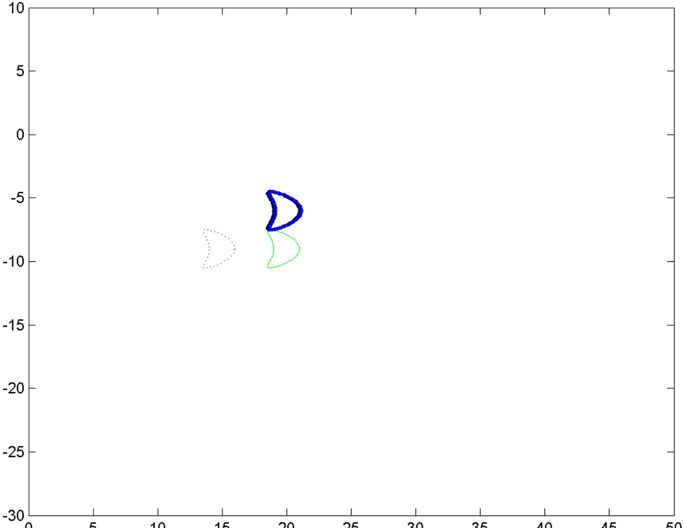

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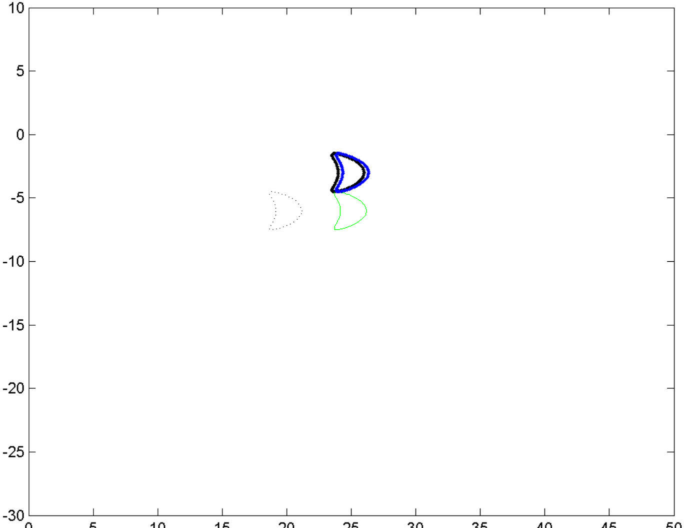

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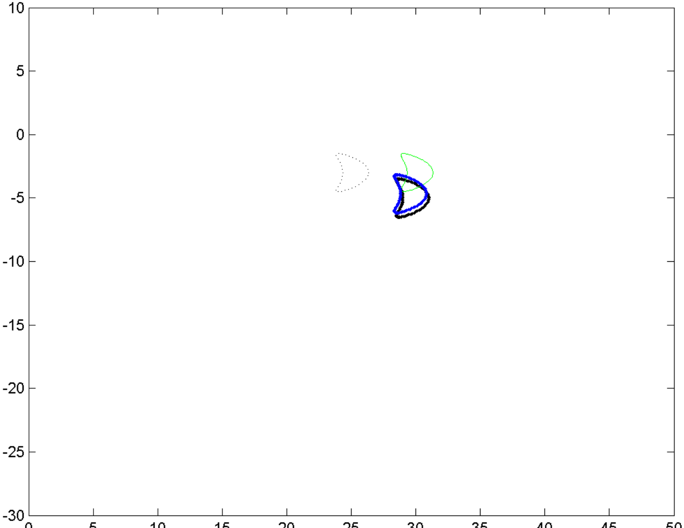

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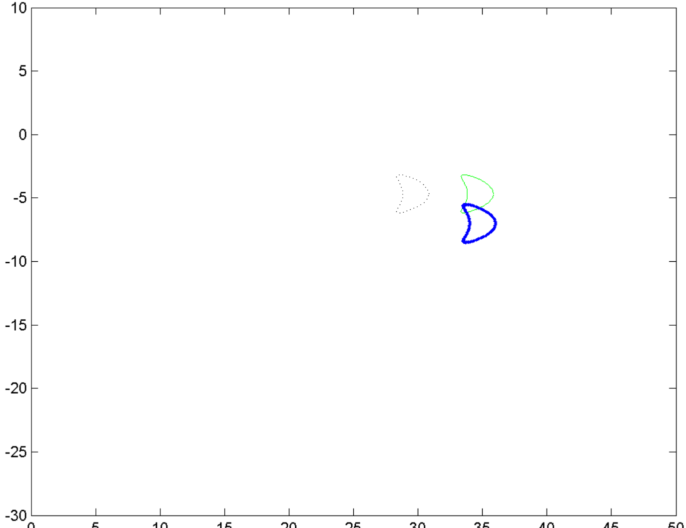

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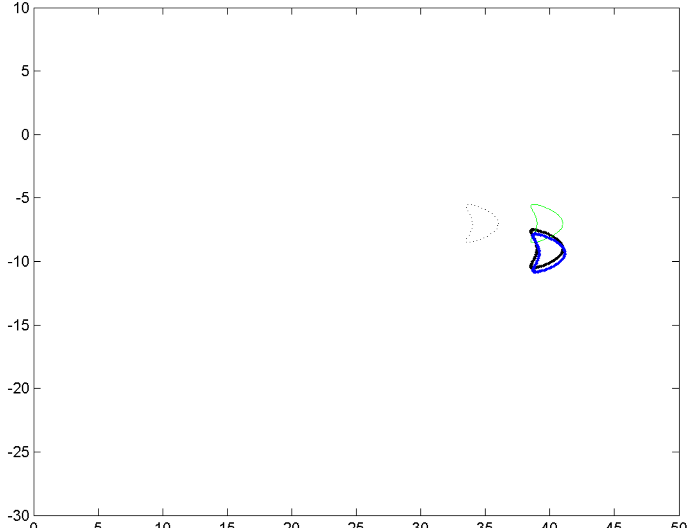

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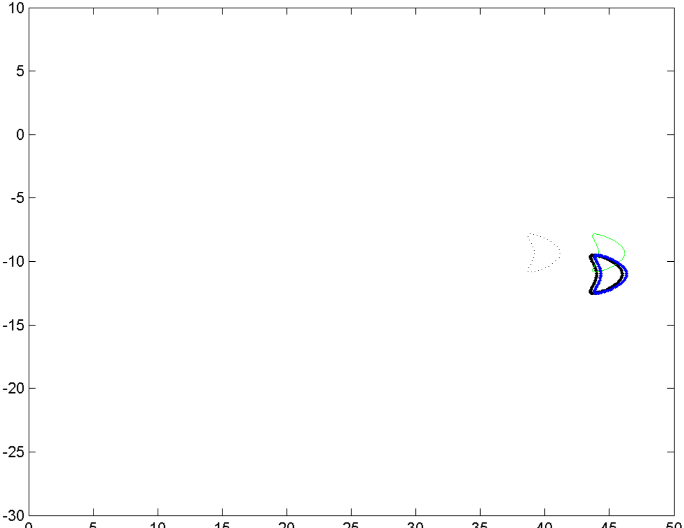
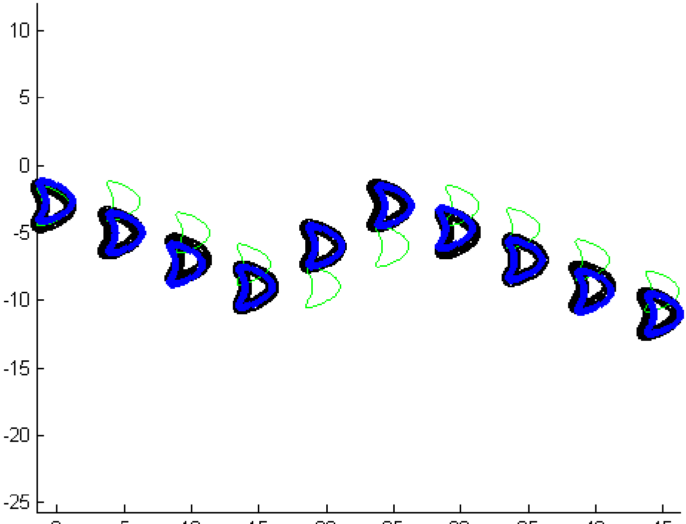
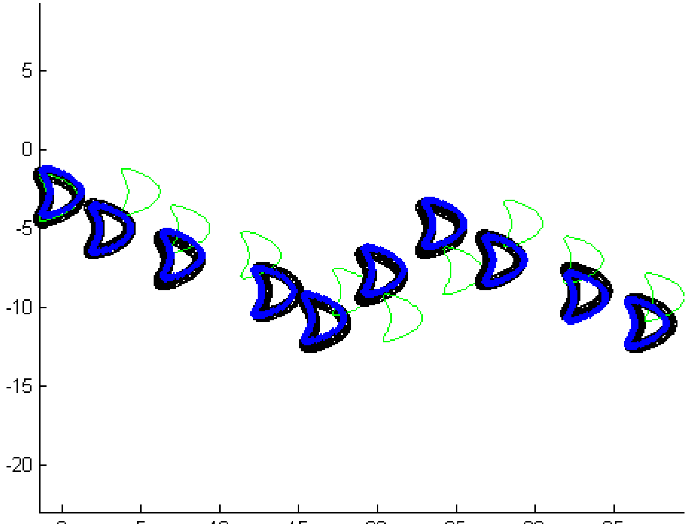


Reconstructed Movement



Reconstructed Movement with random speed



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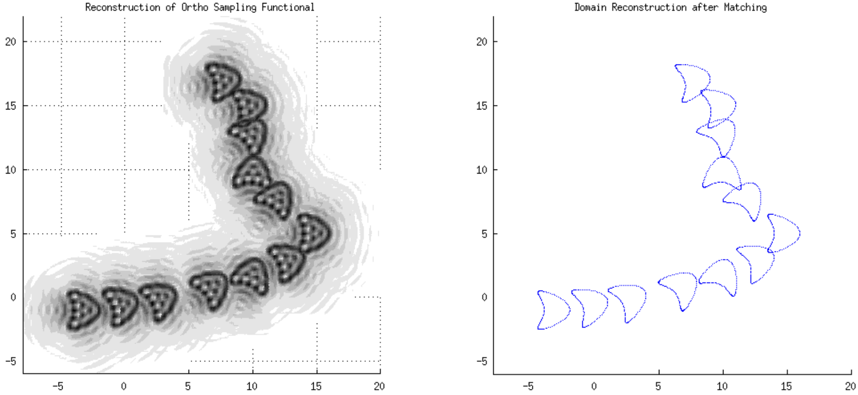
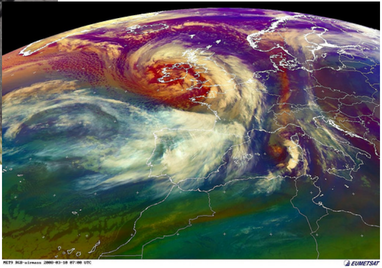


Outline

Weather is Relevant I ...

Plan Travel

Warn and Protect



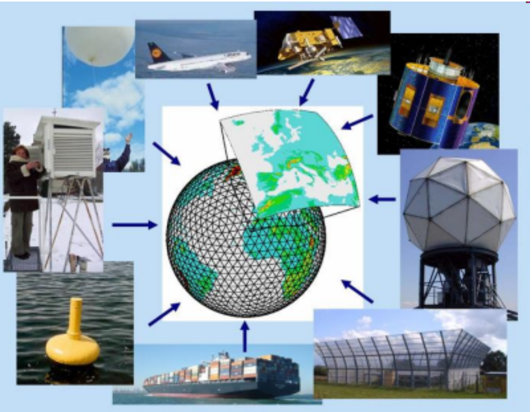
Weather is Relevant II ...

Logistics

Rivers and Environment

Air Control

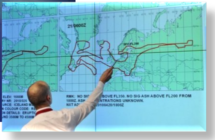
Data Survey ...



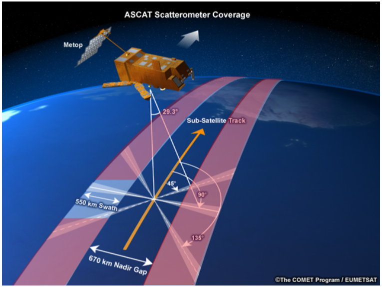
Measured variables include: temperature, moisture, cloud state and coverage, dew point, wind speed and direction, visibility, pressure, weather state, precipitation and snow state and dynamics, sea surface temperature (SST)

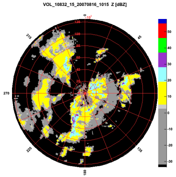
- SYNOP, Ships (ASAP)
- Radiosondes (TEMP, PILOTs),
- Buoys,
- Airplanes (AIREPS, AMDAR, ACARS, ASDAR),
- Radar,
- Wind Profiler,
- Atmospheric Motion Vectors (AMV)
- Scatterometer,
- Radiances (IR, MW),
- GPS/GNSS, Radio Occultations, ZTD, STD,
- Ceilometer,
- Lidar

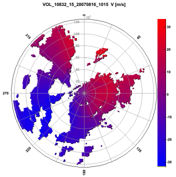
Operational Center with High Performance Computing

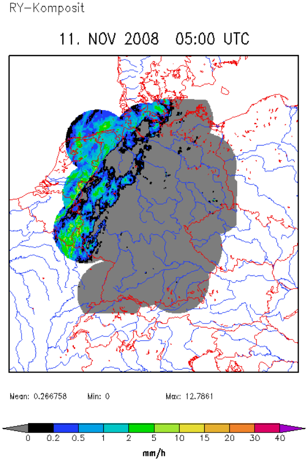


Inverse Scattering as part of Numerical Weather Prediction (NWP)





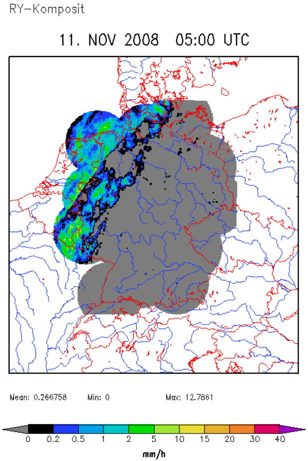




Inverse Scattering in Numerical Weather Prediction (NWP)

Work in Progress:

- Use EnKF for assimilation of radar data.
- Issues with localization are important, error analysis for multistep assimilation.
- Interaction of inversion with system dynamics ...



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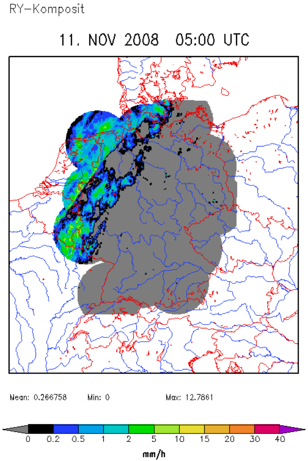

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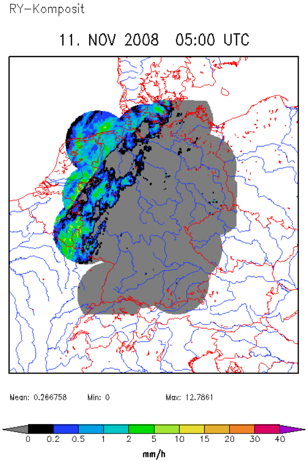

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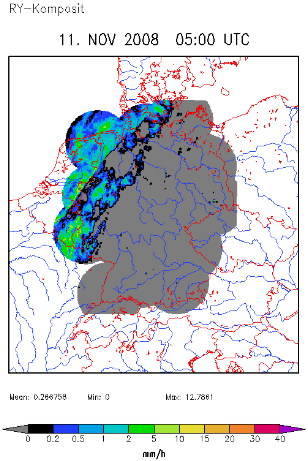

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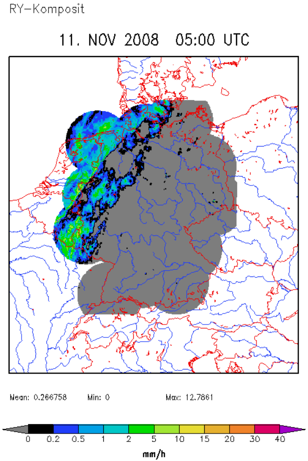

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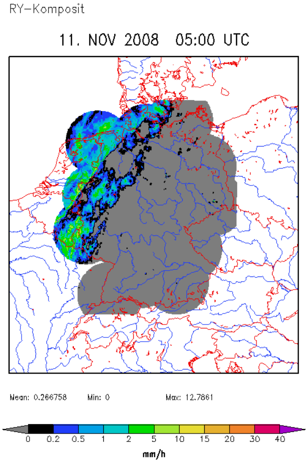

Comments/Further Questions

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- Observability is increased by using the systems dynamics, Control Theory, generic insight and many interesting questions for a particular application area
- Active use/Design of dynamical setup to increase reconstructability!
- Large toolbox of data assimilation methods: variational, ensemble, hybrid ... but not using core inverse scattrering methods!

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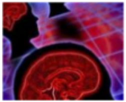
Summer School 2013 and International Symposium 2014

CINN Centre for Integrative Neuroscience and Neurodynamics
EPSRC Pioneering research and skills
Data Assimilation Research Centre
University of Reading



Summer School & Creative Workshop "Data Assimilation and Inverse Problems"
Reading, UK, July 22-26, 2013

Special Issue on "Convective Scale Data Assimilation" Meteorologische Zeitschrift

International Symposium on Data Assimilation 2014
LMU Munich, Germany, Feb 24-28, 2014

http://www.inverseproblems.info/reading:summer_school_2013
http://www.inverseproblems.info/events:isda2014